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Dust Ion Acoustic Solitary Waves in Multi-Ion Dusty Plasma System

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ABSTRACT

A theoretical work has done to observe the existence of dust ion-acoustic (DIA) solitary waves (SWs) in a multi-ion dusty plasma system consisting of inertial positive and negative ions, Maxwell's electrons, and arbitrary charged stationary dust. In this short communication, our research declares that with these components the derivation of Korteweg-de Vries (K-dV) and mixed K-dV (mK-dV) is possible. Here reductive perturbation method has been employed in all these approaches. The first K-dV equation has been derived which gave both bright and dark solitons but for a very limited region. Then the mK-dV equation has been derived that gave bright soliton for a large region, but no dark soliton has been observed.

Keywords: Dusty plasma, Ion acoustic wave, Multi-ion, DIA, Space plasma system, K-dV, and mK-dV.

INTRODUCTION:

Dusty plasma is normal electron-ion plasma with an added highly charged element of small micron or sub-micron sized extremely massive charged gritty (dust grains). Shukla and Silin, (1992) have theoretically shown the low-frequency dust-ion-acoustic waves in a dusty plasma system. Barkan *et al*. (1995) have experimentally verified the existence of dust-ion-acoustic wave in dusty plasma. These waves differ from usual ion-acoustic waves (Lonngren, 1983) due to the conservation of equilibrium charge density $n_{e0}e+n_{d0}Z_{de}-n_{i0}e = 0$, and the strong inequality, $n_{e0} \ll n_{i0}$, where n_{s0} is the particle number density of the species s with s = e (i) d for electrons (ions) dust, Z_d is the figure of electrons residing onto the dust grain side, and e is the magnitude of an electronic charge. DIA wave's linear properties are now prudent understood (Shukla and Mamun, 2002; Barkan *et al*., 1996; and Shukla and Rosenberg, 1999).

The nonlinear structures related with the DIA waves are particularly solitary waves (Bharuthram and Shukla, 1992; Nakamura and Sharma, 2001); shock waves (Nakamura *et al*., 1999; Luo, 2000; and Mamun and Shukla, 2002), etc. These waves have also had a great deal of interest to understand the localized electrostatic perturbations in galactic space (Geortz, 1989; Fortov, 2005), and laboratory dusty plasmas (Shukla and Mamun, 2002; Nakamura and Sharma, 2001, and Barkan *et al*., 1996).

Dusty plasmas create a fully modern interdisciplinary area with direct link to astrophysics, nanoscience, fluid mechanics, and material science as specified through experimental, theoretical, analytical, and arithmetical studies. All of these works (Shukla and Mamun, 2002; Bharuthram and Shukla, 1992; Nakamura and Sharma, 2001; Nakamura *et al*., 1999; Luo, 1995; Mamun, 2009) are limited to planar (1D) geometry and are subjected to some critical value.

A few works have also been done on finite amplitude DIA solitons and shock structures (Luo, 1995), where K-dV or Burgers equations are used, which are not valid because, the latter gives infinitely large amplitude structures which break down the validity of the reductive perturbation method) for a parametric regime corresponding to $A = 0$ or $A \sim 0$ (where A is the coefficient of the nonlinear term of the K-dV or Burgers equation) (Luo, 1995).

Here, $A \sim 0$ means A is not equal to 0, but A is around 0. In our present work, we have been able to show the bright and dark solitons for a large region of multi-ion dusty plasma system in an adiabatic state.

The manuscript is prepared as follows; the model equations are given in Sec. 2, the K-dV equation is derived in Sec. 3, the mK-dV equation is derived in Sec. 4, then results and discussion are given in Sec. 5, and conclusion is given in Sec. 6.

Model Equations

The dynamics of the one-dimensional multi-ion DIA waves are governed by:

$$\partial n_s/\partial t + \partial n_s u_s/\partial x = 0, \quad (1)$$

$$\partial u_i/\partial t + u_i \partial u_i/\partial x = -\partial\psi/\partial x - (\delta_i/n_i)(\partial p_i/\partial x), \quad (2)$$

$$\partial u_n/\partial t + u_n \partial u_n/\partial x = 1/\mu(\partial\psi/\partial x - \delta_n/n_n\, \partial p_n/\partial x), \quad (3)$$

$$e\partial\psi/\partial x - 1/n_e\, \partial p_e/\partial x = 0, \quad (4)$$

$$\partial p_s/\partial t + u_s \partial p_s/\partial x + \gamma p_s\, \partial u_s/\partial x = 0, \quad (5)$$

$$\partial^2\psi/\partial x^2 = [1-\mu_n + \mu_d]\, \exp^{\psi} + \mu_n n_n - \mu_d - n_i, \quad (6)$$

where n_s is the number density with $s = n(i)e(d)$ of negative ion (positive ion) electron (stationary dust), u_s is the fluid speed of s, m_j is the positive (when $j = i$) or negative (when $j = n$) ion mass, Z_d is the number of electron occupy on the dust grain side, e = magnitude of the electron-charge (q), ϕ is the electrostatic wave potential; n_{s0}, (n_{j0}), and n_{d0} are the equilibrium value of n_s, (n_j), and n_d respectively i.e. n_s, (n_j), and n_d are the number density normalized by n_{s0}, (n_{j0}), and n_{d0} respectively, p_i is the pressure of species i, γ is an adiabatic index, x is the space variable, and t is the time variable.

K-dV Equation

For the DIA K-dV equation we introduce the stretched coordinates:

$$\zeta = \epsilon^{1/2}(x - V_p t), \text{ and } \tau = \epsilon^{3/2} t, \quad (7)$$

Where, V_p is the wave phase speed (ω/k), and ϵ is a smallness parameter ($0 < \epsilon < 1$).

To get the dispersion relation, we expand n_s, u_s, p_s, and ϕ with s be the charged species like positive and negative ion, electron in power series of ϵ, to their equilibrium and perturbed parts,

$$n_s = 1 + \epsilon n_s^{(1)} + \epsilon^2 n_s^{(2)} + \epsilon^3 n_s^{(3)} + \cdots, \quad (8)$$

$$u_s = 0 + \epsilon u_s^{(1)} + \epsilon^2 u_s^{(2)} + \epsilon^3 u_s^{(3)} + \cdots, \quad (9)$$

$$p_s = 0 + \epsilon p_s^{(1)} + \epsilon^2 p_s^{(2)} + \epsilon^3 p_s^{(3)} + \cdots, \quad (10)$$

$$\psi = 0 + \epsilon\psi^{(1)} + \epsilon^2\psi^{(2)} + \epsilon^3\psi^{(3)} + \cdot\cdot, \quad (11)$$

Where $n_s^{(1)}$, $u_s^{(1)}$, $p_s^{(1)}$, and $\psi^{(1)}$ are the perturbed part of n_s, u_s, p_s, and ψ respectively.

Combining above equations, we get -

$$V_p = (-b \pm \sqrt{(b2 - 4ac)}/2a)^{(1/2)}, \quad (12)$$

Where,

$\alpha = (1-\mu_n + \mu_d)$, $a = \mu\alpha$, $b = \mu_n + \mu - \alpha\gamma\delta_n - \alpha\mu\gamma\delta_i$, and $c = \alpha\gamma^2\delta_n\delta_i - \gamma\delta_n - \gamma\delta_i\mu_n$.

Equation (12) represents linear dispersion relation.

The next higher order of ϵ can be simplified as an equation of the form:

$$\partial\psi/\partial\tau + A\psi\, \partial\psi/\partial\zeta + \beta\partial^3\psi/\partial^3\zeta = 0, \quad (13)$$

Where,

$A = Y/X$,

$\beta = 1/X$,

$X = (2V_p/d_2^2)(2V_p\mu\mu_n/d_1)$,

$Y = -c_1/d_2^3 + \mu_n c_2/d_1^3 + \mu_n c_3/d_1^3$,

$c_1 = V_p^4 + 2\,V_p^2\gamma\delta_i - 3V_p^2 + \gamma^2\delta_i^{\,2} - 2\gamma\delta_i + \gamma^2\delta_i$,

$c_2 = V_p^4\mu^2 - 2\mu\,V_p^2\gamma\delta_n + 3\mu V_p^2$,

$c_3 = \gamma^2\delta_n^2 + 2\gamma\delta_n - \gamma^2\delta_n$.

Equation (13) is known as K-dV equation. We get stationary localized solution of (13) by introducing a transformation $\xi = \zeta - U_{0\tau}$;

$$\psi = \psi_m \mathrm{sech}^2\,[(\zeta - U_{0\tau})/\delta], \qquad (14)$$

Where the amplitude ψ_m and the width δ are given by $\psi_m = 3U_0/A$, and $\delta = \sqrt{4\beta/U_0}$, respectively.

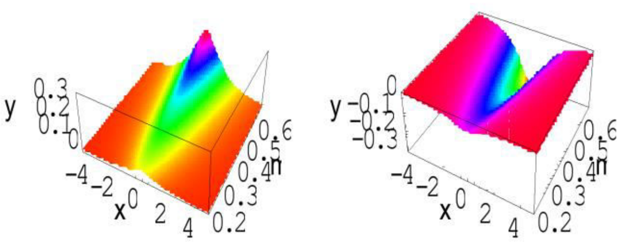

Fig 1: Bright and dark K-dV solitons.

Equation (14) is the solution of K-dV equation. This represents a solitary wave. We observed that the **Fig 1** shows the existence of bright and dark K-dV solitons with mass number density (μ).

mK-dV Equation

For the third order calculation a new set of stretched coordinates is applied:

$$\zeta = \epsilon(x - V_p t),\ \tau = \epsilon^3 t, \qquad (15)$$

Using (15) we can find the same values of $n_i^{(1)}$, $n_n^{(1)}$, $n_e^{(1)}$, $u_i^{(1)}$, $u_e^{(1)}$, $u_n^{(1)}$, $p_i^{(1)}$, $p_e^{(1)}$, $p_n^{(1)}$, and V_p as like as that in K-dV.

To the next order approximation of ϵ, we obtain a set of equations, which, after using the values $n_i^{(1)}$, $n_n^{(1)}$, $n_e^{(1)}$, and V_p, can be simplified and applying the condition, $\psi \neq 0$ (so, it's coefficient is zero), we get,

$$\tfrac{1}{2}\{A(\psi^{(1)})^2\} = 0 \qquad (16)$$

For the next higher order of ϵ, we obtain an equation:

$$\partial\psi/\partial\tau + \alpha\beta\psi^2\partial\psi/\partial\zeta + \beta\partial^3\psi/\partial\zeta^3 = 0, \qquad (17)$$

Where,

$\alpha = F(-a_1^2 + 15/2 - 21\gamma\delta_n/2a_1 - 5\gamma^2\delta_n/2a_1$
$-3\gamma^3\delta^2_n/a_1^2) - F(3\gamma^2\delta_n^2/a_1^2) + G(a_2^2 - 15/2$
$-21\gamma\delta_i/2a_2) - G(5\gamma^2\delta_i/2a_2 - 3\gamma^3\delta_i^{2/}a_2^2 - 3\gamma^2\delta_i^2/a_2^2)$,

$\beta = V_p a_1^2 a_2^2/(-2\mu\mu_n a_2^2\,V_p^2 - 2\gamma\delta_i a_1^2)$,

Where,

$F = \mu_n/a_1^3$, $G = 1/a_2^3$, $a_1 = (\gamma\delta_n - \mu V_p^2)$, and $a_2 = (V_p^2 - \gamma\delta_i)$.

Equation (17) is known as mK-dV equation. The stationary localized solution of (17), obtained by introducing a transformation $\xi = \zeta - U_0\tau$, is, therefore, directly given by

$$\psi = \psi_m \mathrm{sech}\,[\xi/\Delta], \qquad (18)$$

Where the amplitude ψ_m and the width δ are given by $\psi_m = \sqrt{6U_0/\alpha\beta}$, and $\delta = 1/\psi_m\sqrt{\gamma}$. where the amplitude ψ_m and the width Δ are given by $\psi_m = \sqrt{6U_0/\alpha\beta}$, $\Delta = 1/(\sqrt{\gamma}\psi_m)$, and $\gamma = \alpha/6$.

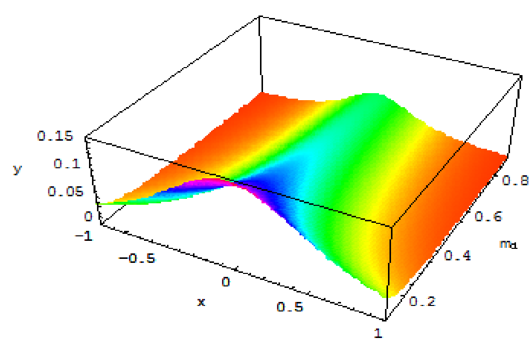

Fig 2: Bright mK-dV soliton.

Fig 2 gives the existence of mK-dV solitons. We get only one type of solitons, bright solitons.

RESULTS AND DISCUSSION:

Dust ion acoustic K-dV and mK-dVsolitons have been investigated in a multi-ion dusty plasma system where we observed;

1. The positive and negative K-dVsolitons are observed.
2. The width and amplitude of the K-dVsolitons varies with polaritychanges.
3. Existence of positive mK-dVsolitons is observed.

CONCLUSION:

Present investigation are valid for tiny amplitude DIA K-dV, and mK-dV solitons. The first K-dV equation has been derived which gave both bright and dark solitons but for a very limited region. Though we have considered positive and negative ions, Maxwell's electrons, and arbitrarily charged stationary dust, our model is applicable for small amplitude waves only.

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CONFLICTS OF INTEREST:

The authors declare that they have no competing interests with respect to the research.

REFERENCES:

1. Barkan A., Angelo N. D, and Merlino R. L., (1996). "Experiments on ion-acoustic waves in dusty plasmas," *Planet. Space Sci*., **44**, pp. 239-242. https://doi.org/10.1016/0032-0633(95)00109-3
2. Barkan A., Merlino R. L. and Angelo N. D, (1995). "Laboratory monitoring of the dust-acoustic wave mode," *Phys. Plasmas*, **21**, pp. 3563. https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.1040.4162&rep=rep1&type=pdf
3. Barkan A., Angelo N. D and Merlino R. L., (1996). "Experiments on ion-acoustic waves in dusty plasmas," *Planet. Space Sci*., **44**, pp. 239. https://doi.org/10.1016/0032-0633(95)00109-3
4. Bharuthram R. and Shukla P. K., (1992). Large amplitude ion-acoustic solitons in dusty plasma, *Planet. Space Sci.*, **40**, pp. 973. https://doi.org/10.1016/0032-0633(92)90137-D
5. Bliokh, P., Sinitsin, V. and Yaroshenko, V. (1995). Dusty and Own-Gravitational Plasmas in Space. Dordrecht, Netherlands: Kluwer. https://link.springer.com/content/pdf/bfm%3A978-94-015-8557-6%2F1.pdf
6. Fortov V. E., (2005). Basics of dusty plasma, *Phys. Rep*., **31**(1), pp 46-56.
7. Geortz C. K., (1989). Dusty plasmas in the solar system, *Rev. Geophys*., **27**, pp. 271. https://doi.org/10.1029/RG027i002p00271
8. Lonngren K. E., (1983). "Soliton experiments in plasmas," *Plasma Phys*., **25**, pp. 943-982.
9. Luo Q. Z., (2000). Ion acoustic shock formation in converging magnetic field geometry, *Phys. Plasmas*, 7, pp. 3457. https://doi.org/10.1063/1.874074
10. Mamun A A and Shukla P. K., (2002). Cylindrical, and spherical dust ion–acoustic solitary waves, *IEEE Trans. Plasma Sci*., **30**, pp. 720. https://doi.org/10.1063/1.1458030
11. Mamun A. A., (2009). "Solitary waves and double covers in a dusty electronegative plasma", *Phys. Rev. E*, **80**, pp. 046406. https://doi.org/10.1103/PhysRevE.80.046406
12. Nakamura Y., Bailung H., and Shukla P. K., (1999). "Observation of ion-acoustic shocks in a dusty plasma," *Phys. Rev. Lett*., **83**, pp. 1602-1605. https://doi.org/10.1103/PhysRevLett.83.1602
13. Nakamura Y. and Sharma A., (2001). NMR study of the magnetic properties of the quasi-one-dimensional conductor (TMTTF), *Phys. Plasmas*, **8**, pp. 3931.
14. Shukla P. K. and Rosenberg M., (1999). "Boundary effects on dust-ion-acoustic and

dust-acoustic waves in collisional dusty plasmas," *Phys. Plasmas*, **6**, pp. 1038. https://doi.org/10.1063/1.873345

15. Shukla P. K., and Silin V. P., (1992). "Dust ion acoustic wave," *Phys. Scripta*, **45**, p. 508. https://doi.org/10.1088/0031-8949/45/5/015
16. Shukla P. K. and Mamun A. A., (2002). "Introduction to Dusty Plasman Physics," Institute of Physics Publishing Ltd. Bristol, ISBN 075030653X.
17. Verheest, F. (2002). Waves in Dusty Space Plasmas. Dordrecht, Netherlands: Kluwer. https://www.springer.com/gp/book/9781402003738

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